Bridging Units: Resource Pocket 9

Vectors

Vectors are not listed in the current or new Programmes of Study at Key Stage 3 although students may be familiar with the concept of vectors from physics / science lessons.

The new Foundation tier GCSE specification involves addition and subtraction of vectors and multiplication of vectors by a scalar. Diagrammatic and column vector representation is also required at Foundation tier, whilst the use of vectors to construct geometrical arguments and proofs is required at Higher tier.

Therefore, being aware of the basics of vectors and how they are represented would be highly desirable prior to commencing a GCSE course.

The activities form a mainly basic introduction to vectors focusing on a visual approach.

The resources are suitable for students progressing to the Foundation tier apart from Developing Understanding 3 and Skills Builder 3 which are quite demanding and are only suitable for those progressing to the Higher tier.

The resource pocket progresses through three sections: developing understanding, skills builders and problem solving activities. As with all 9 resource pockets there are a number of different learning styles and approaches used to cater for a variety of learners.

1 Developing Understanding

These are class based, teacher led questions with suggested commentary to get the most from a class or small group discussion. The boxed text can either be copied onto the whiteboard for class discussion, or printed onto cards and handed out to students to be used for paired or small group work.

2 Skills Builders

These are standard progressive worksheets that can be used to drill core skills in a particular area.

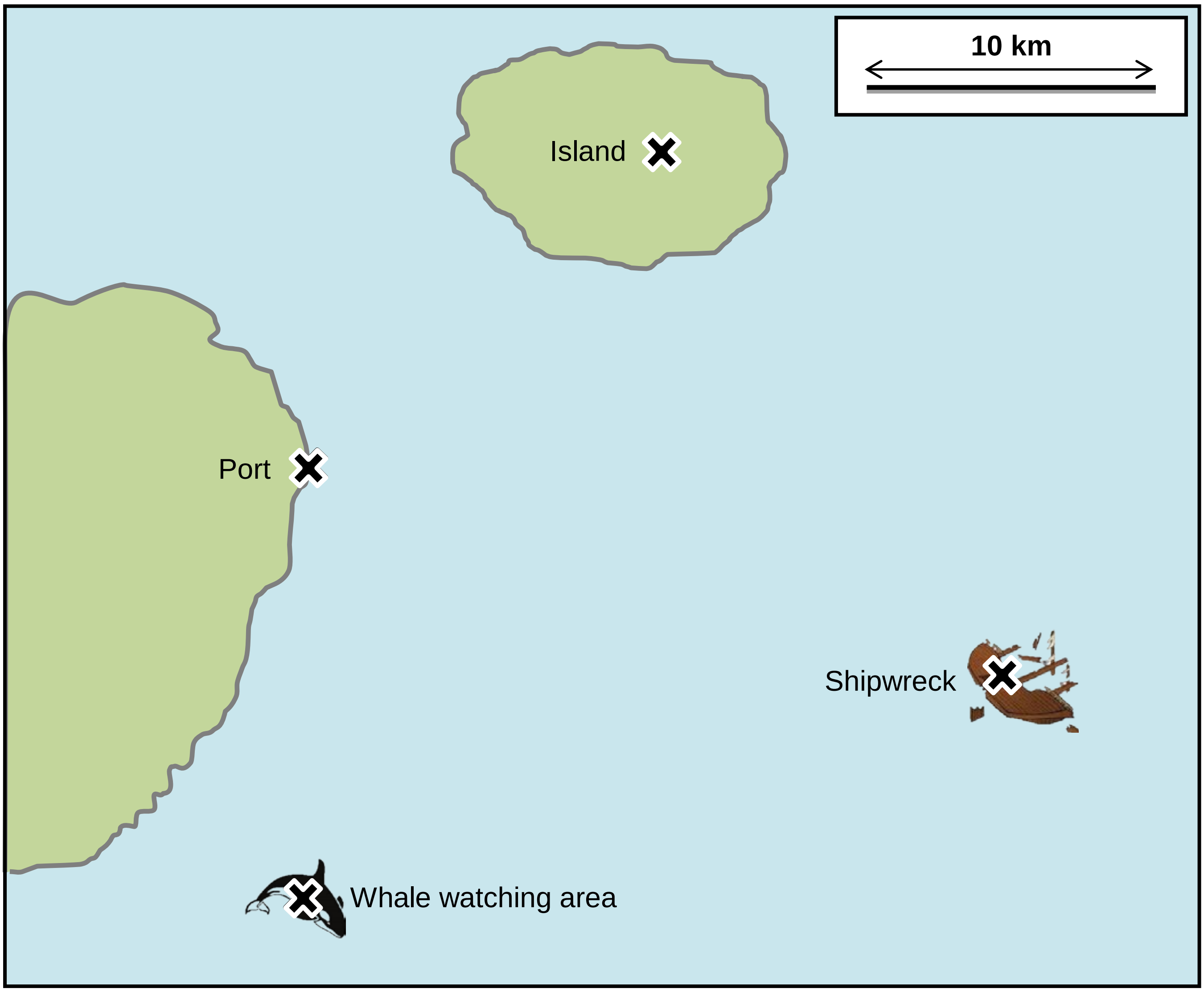
3 Problem Solving Activities

Extension activities for paired work or small group work to develop problem solving skills whilst focussing on a particular area of mathematics that students can learn to apply.

Developing Understanding 1

This section introduces students to the idea of a vector and how these might be represented.

Display the diagram in the box below.



Ask students:

- If a ship wanted to sail from the port to the island, how might we describe the route?
- What about if a ship wanted to sail from the shipwreck to the port – how might we describe the route?
- How would we describe the route from the whale watching area to the port?

Discuss ideas with the students. Incorporate comments relating to distance and direction, perhaps by writing two lists on the board of any words that students mention when describing their routes. Key words might include kilometres, north-west, south-east etc, degrees, bearings. For example, travelling from the port to the island students might say:

“Travel approximately 15km in a north-east direction”.

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Explain to students that to provide clear sailing directions we need to provide both **a distance and a direction**. The route can be shown on a map using an arrow, which we call a vector. Explain that vectors can be described in two different ways and then display the following box.

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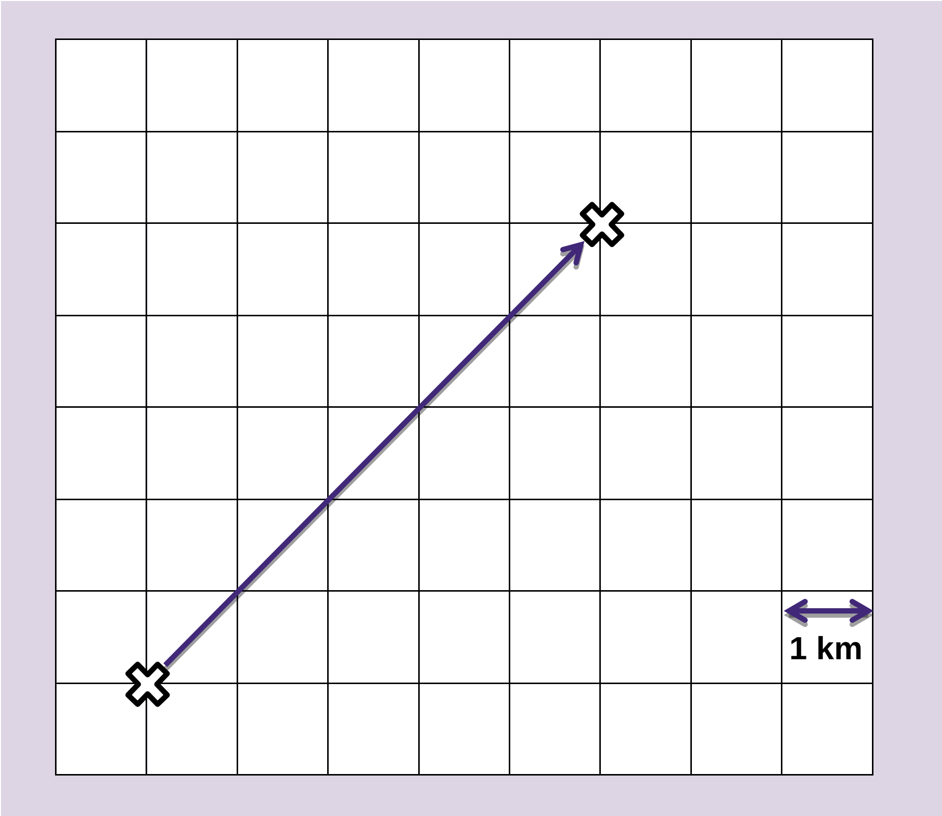

How could we define this vector? Allow students time to discuss this in pairs before sharing their ideas. Responses are likely to fall into two categories:

- 5 to the right and 5 up (or equivalent)
- 7.1km (approximately) in a north-east direction

If students have not yet studied Pythagoras Theorem they might draw a scale diagram to calculate the distance 7.1km (the exact distance is $5\sqrt{2}$ km).

The first category will be discussed in the remainder of this section. (See Developing Understanding 2 for more discussion of the second category).

Explain to students that the '5 to the right and 5 up' can be described in terms of a vector which is written as:

$$\begin{pmatrix}5\\5\end{pmatrix}\begin{array}{l}\longleftarrow \textbf{movement to the right}\\ \longleftarrow \textbf{movement up}\end{array}$$

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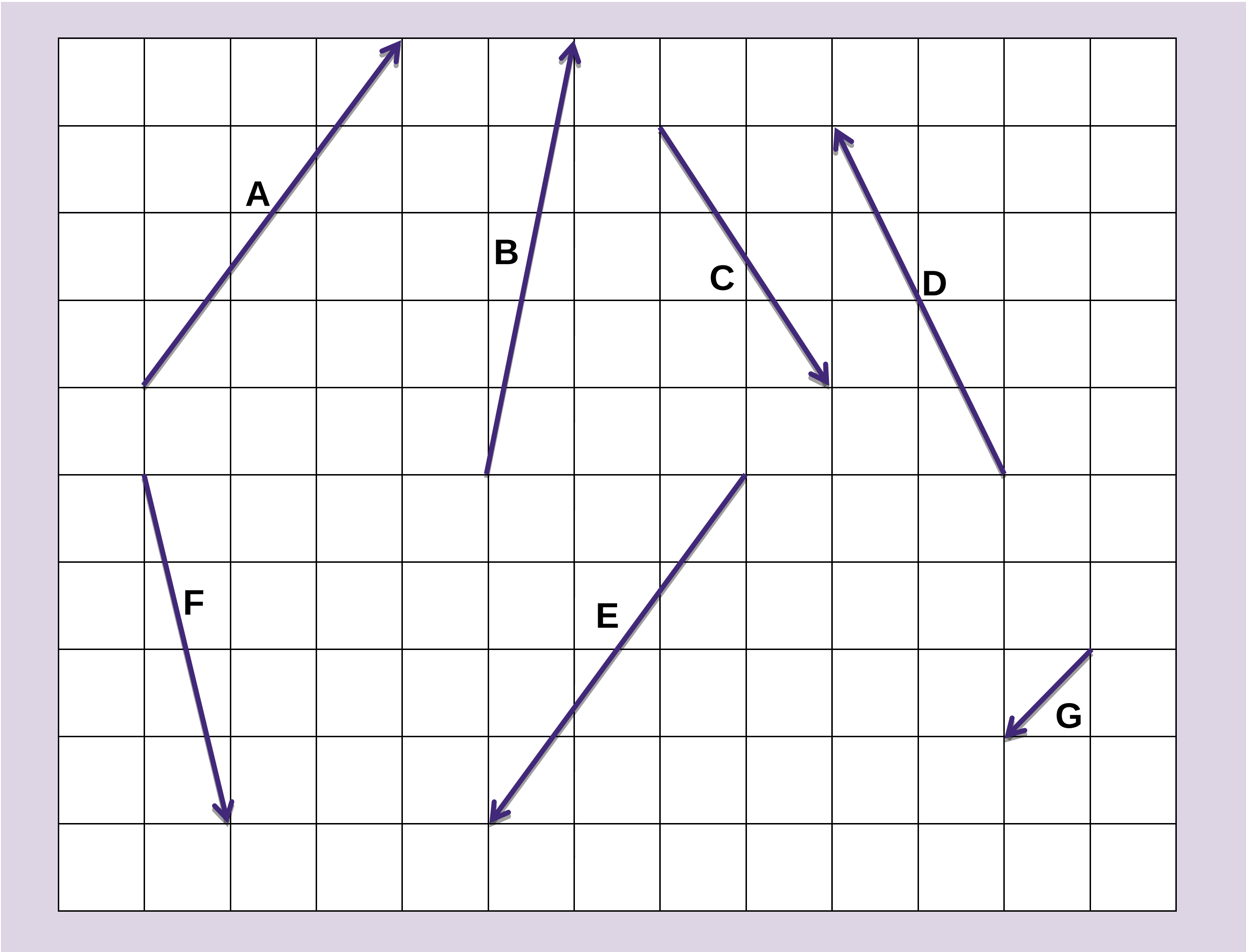
Now show students the box below and ask them to describe each line in terms of a vector. Do not explain to students how to represent 'left' or 'down' at this stage – see if they can identify the need to use negative numbers.



The answers are:

$$A = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \quad B = \begin{pmatrix} 1 \\ 5 \end{pmatrix} \quad C = \begin{pmatrix} 2 \\ -3 \end{pmatrix} \quad D = \begin{pmatrix} -1 \\ 4 \end{pmatrix} \quad E = \begin{pmatrix} -3 \\ -4 \end{pmatrix} \quad F = \begin{pmatrix} 1 \\ -4 \end{pmatrix} \quad G = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

Ensure students are clear on the fact that negative numbers represent movement to the left (top digit of the vector) or down (bottom digit of the vector).

Ask students if they have spotted any vectors that have something in common. Hopefully they will have noticed that vectors D and F and vectors A and E are parallel; students might also identify that the vectors are going in opposite directions and are the negative of each other.

The use of column vectors is consolidating in Skills Builder 1.

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Developing Understanding 2

Explain that in this section we will look at the other way to describe vectors, which is by stating a distance and a direction. This is one of the ways identified in Developing Understanding 1.

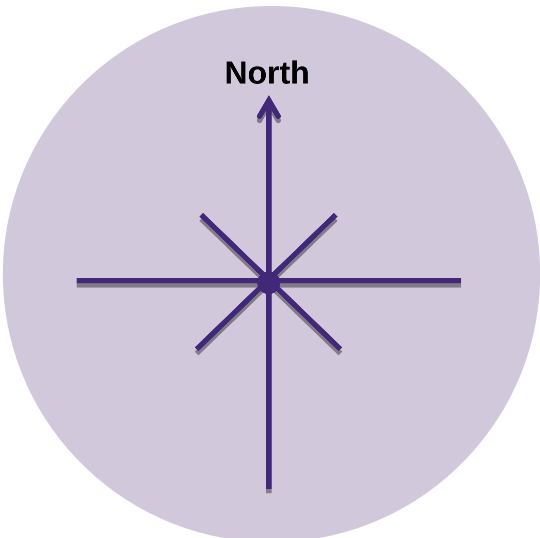
Show students the diagram below and ask them to suggest ways of defining the direction to each of Burnslee and Clarville from Ashberry.

Likely responses include ‘around north east’ or ‘just past south’. If students have met bearings before they may suggest more precise definitions such as ‘a bearing of around 200°’.

Explain that an exact method is needed to identify location and this involves measuring precise angles and distances. Show students the box below and explain the three key features when defining a bearing:

- The angle is measured clockwise
- The angle is measured from a North line which must be drawn in
- The angle must be given in three digits (inserting a 0 at the beginning if the angle has only two digits

Together, agree the three-figure bearings for N, NE, E, etc.…



The answers are:

- North 000°
- North East 045°
- East 090°
- South East 135°
- South 180°
- South West 225°
- West 270°
- North West 315°°

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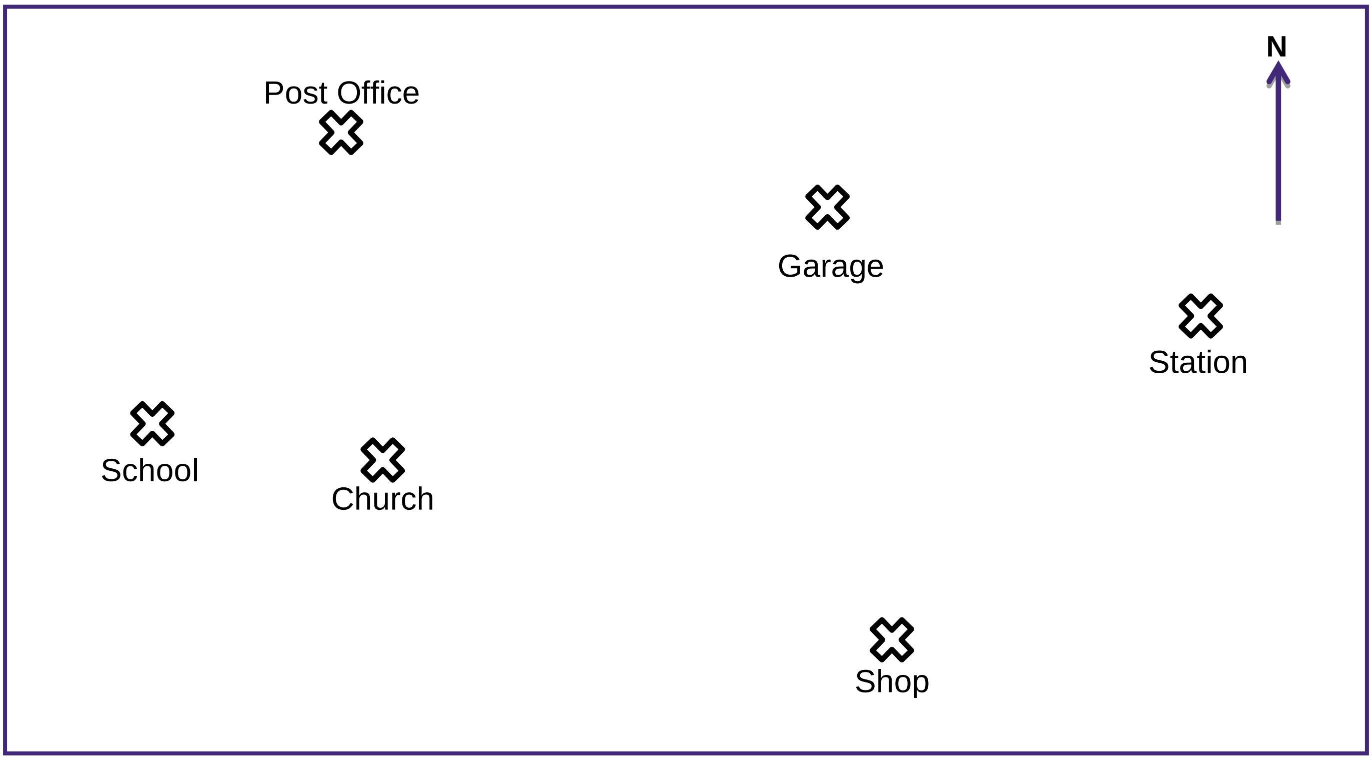
Now show students the next box, which represents key locations in a small town – students will also need a copy of the diagram on paper, so that they can measure the angles.

Ask students to state the bearings of each location **from the Church**. Advise students that we draw the North line at the location which the bearing is required 'from', ie for the bearing of A **from** B, draw the North line **at B**.



Once the bearings have been correctly identified, tell students that 1 cm on the diagram represents 0.2 km.

Ask students to write the facts in the box above as full sentences, including a bearing and a distance.

eg B is 5 km from A on a bearing of 235°.

The full answers are:

- Garage 1.34 km from the Church on a bearing of 061°
- Station 2.2 km from the Church on a bearing of 081°
- Shop 1.44 km from the Church on a bearing of 110°
- School 0.64 km from the Church on a bearing of 260°
- Post Office 0.88 km from the Church on a bearing of 353°

As an extension activity, one student writes a statement but misses out a key fact and their partner must complete the sentence using the diagram. They should use a different starting point other than the Church.

For example

____________________ is 1.16 km from the garage on a bearing of 173°

The Post Office is 2.36 km from the station on a bearing of ________________ °

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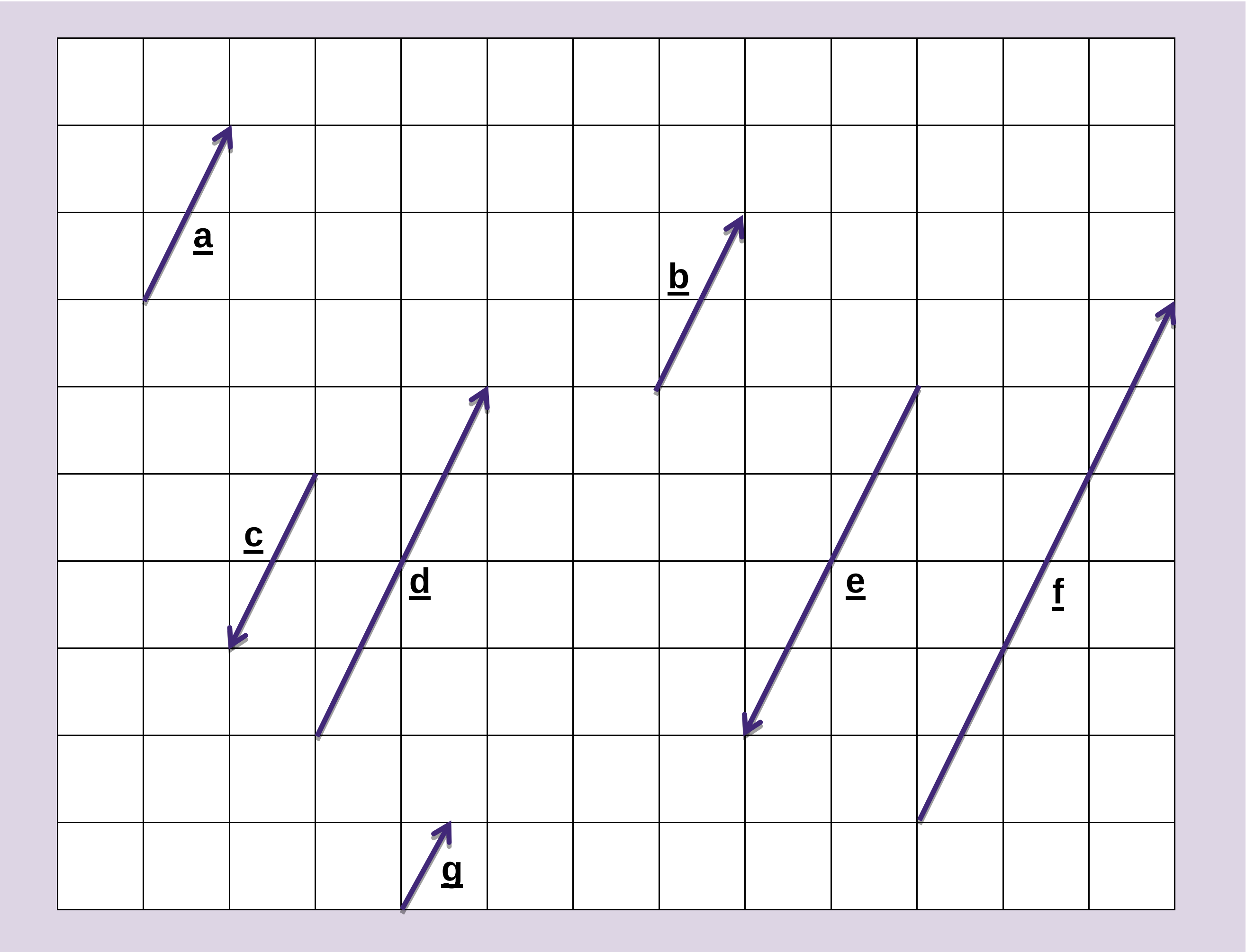
Developing Understanding 3

This section is suitable for students progressing to the Higher tier at GCSE.

Explain that sometimes we do not want to know the exact vector between two locations and instead represent vectors using diagrams which are not drawn to scale. In these cases, it is common to use algebra to represent the vectors.

Display the box below and ask the following questions:

- How are each of the vectors linked to vector $\underline{\mathbf{a}}$?
- Can the other vectors be expressed algebraically in terms of the vector $\underline{\mathbf{a}}$?
- What do all of the vectors have in common?



The answers are

- $\underline{\mathbf{b}} = \underline{\mathbf{a}}$, $\underline{\mathbf{c}} = -\underline{\mathbf{a}}$, $\underline{\mathbf{d}} = 2\underline{\mathbf{a}}$, $\underline{\mathbf{e}} = -2\underline{\mathbf{a}}$, $\underline{\mathbf{f}} = 3\underline{\mathbf{a}}$, $\underline{\mathbf{g}} = 0.5\underline{\mathbf{a}}$
- All the vectors are parallel to each other.

Point out that we tend to underline vectors to indicate that they are vectors and not just algebraic variables. In textbooks vectors are usually written in bold.

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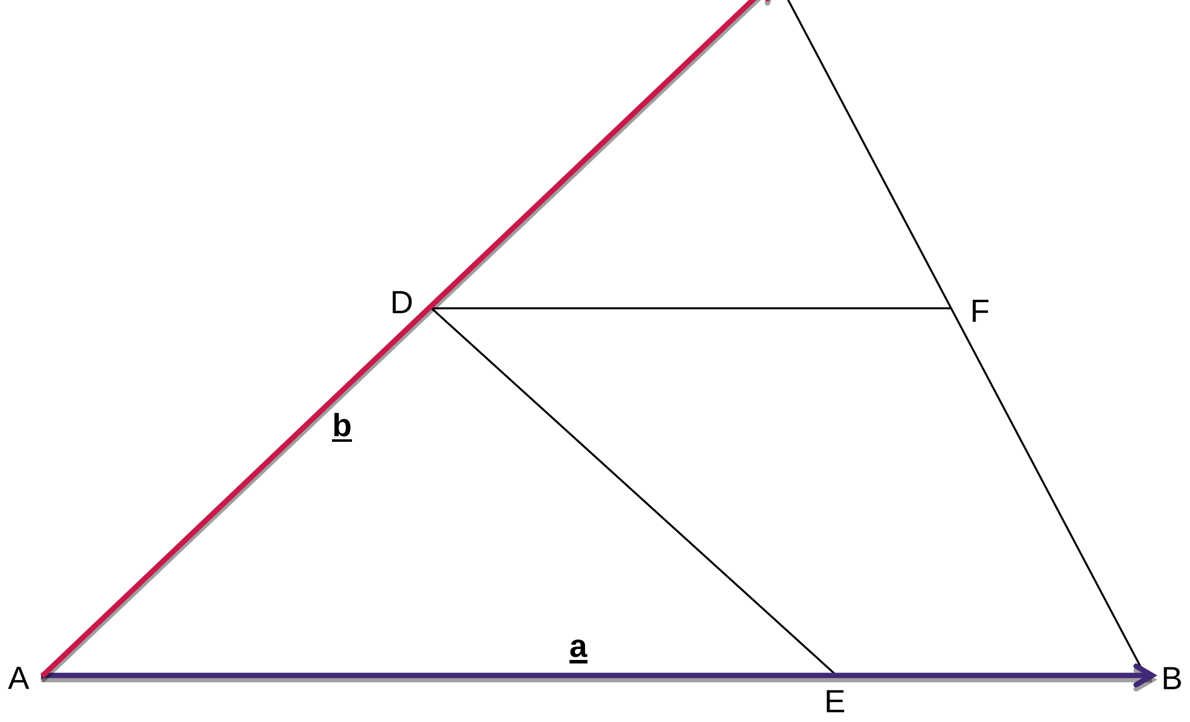
Now display the following box:

D is the midpoint of the side AC.

E is two-thirds along the line from A to B.



Explain that the vector **a** is represented on the diagram along the side AB and the vector **b** along the side AC.

Ask students how we could represent the following vectors in terms of **a** and **b**. Point out that the order of the letters is important. The arrow above the two letters indicates 'the vector between' eg $\overrightarrow{AB}$ represents the vector from A to B

- $\overrightarrow{BA}$
- $\overrightarrow{AD}$
- $\overrightarrow{CA}$
- $\overrightarrow{CB}$
- $\overrightarrow{AE}$
- $\overrightarrow{CE}$
- $\overrightarrow{DE}$

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- $\overrightarrow{BA} = -\underline{\mathbf{a}}$
- $\overrightarrow{AD} = \frac{1}{2}\underline{\mathbf{b}}$
- $\overrightarrow{CA} = -\underline{\mathbf{b}}$
- $\overrightarrow{CB} = -\underline{\mathbf{b}} + \underline{\mathbf{a}}$ or $\underline{\mathbf{a}} - \underline{\mathbf{b}}$
- $\overrightarrow{AE} = \frac{2}{3}\underline{\mathbf{a}}$
- $\overrightarrow{CE} = -\underline{\mathbf{b}} + \frac{2}{3}\underline{\mathbf{a}}$ or $\frac{2}{3}\underline{\mathbf{a}} - \underline{\mathbf{b}}$
- $\overrightarrow{DE} = -\frac{1}{2}\underline{\mathbf{b}} + \frac{2}{3}\underline{\mathbf{a}}$ or $\frac{2}{3}\underline{\mathbf{a}} - \underline{\mathbf{b}}$

Point out that some of the vectors require us to travel 'via' another vertex on the diagram.

Skills Builder 3 offers the opportunity to practice more of this type of question. Students could work individually or in pairs on these questions. The questions are quite difficult so are probably suitable only for more able students who are progressing to the Higher tier and could be used as an extension activity.

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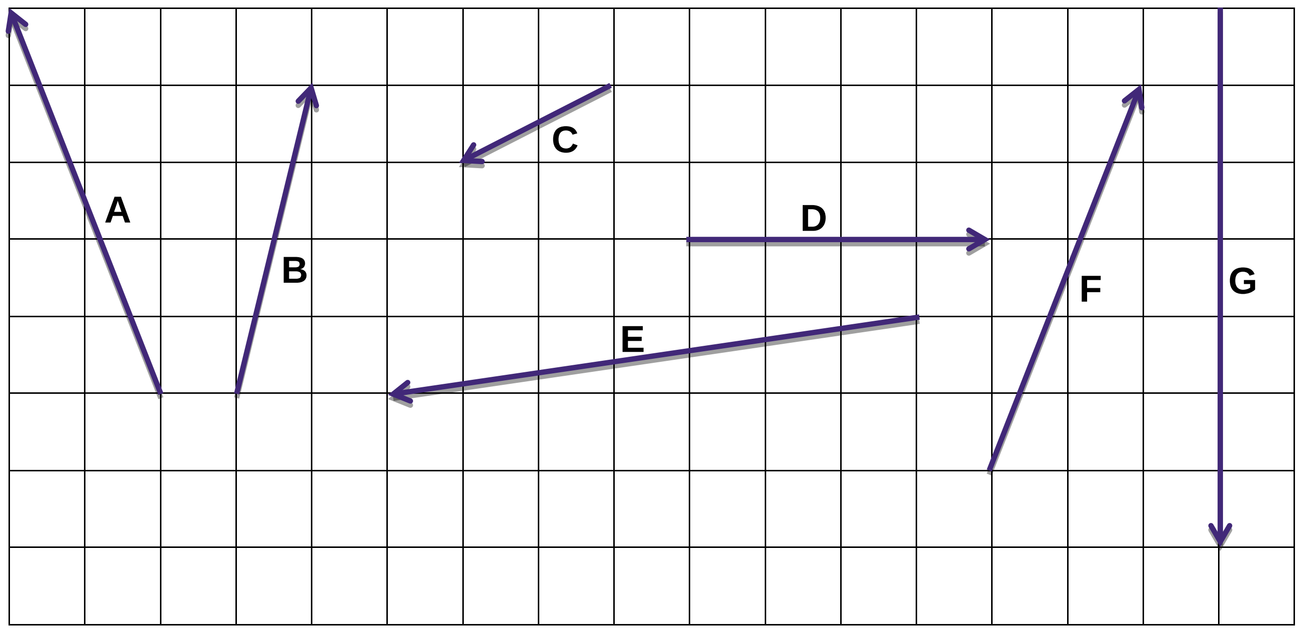
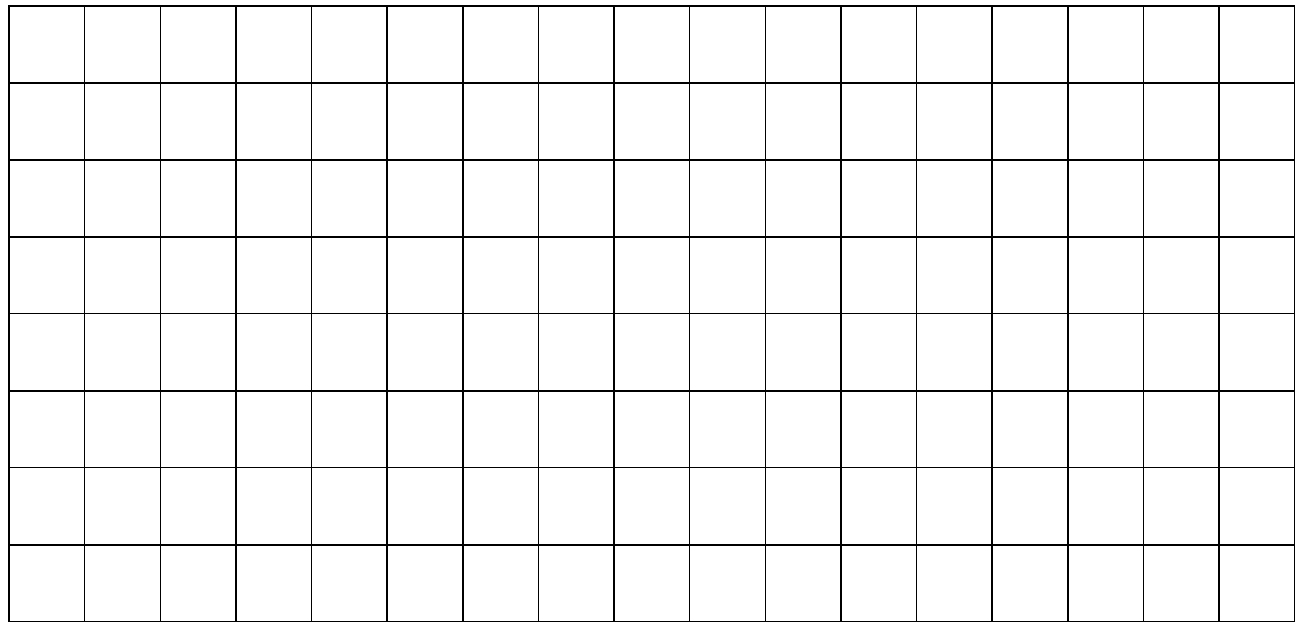
Skills Builder 1: Column Vectors

1 Write each of the vectors on the grid below in the column vector format.



2 On the grid below draw the vectors.

$A = \begin{pmatrix} 3 \\ -5 \end{pmatrix}$ $B = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$ $C = \begin{pmatrix} -2 \\ -5 \end{pmatrix}$ $D = \begin{pmatrix} -2 \\ 4 \end{pmatrix}$ $E = \begin{pmatrix} 0 \\ 5 \end{pmatrix}$



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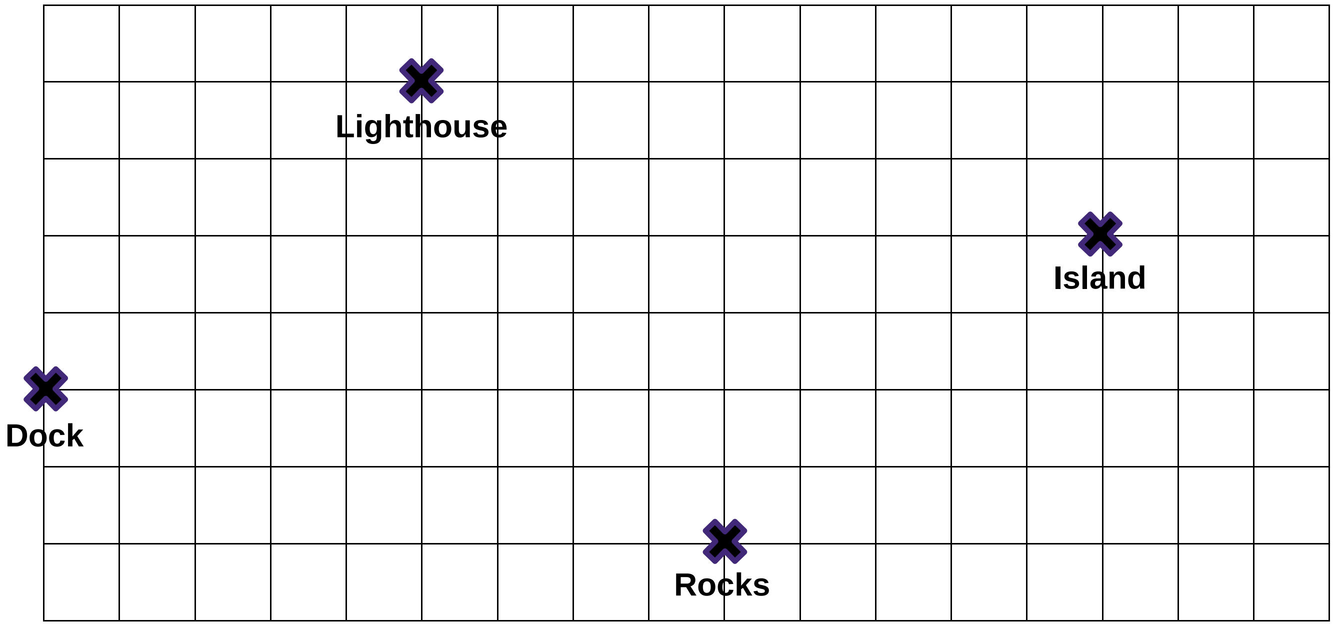
3 A boat sets sail from a dock and travels to the lighthouse, then on to the island and finally to the rocks, before heading back to the dock.



(a) Write down the vector for the section of the journey from:

(i) the dock to the lighthouse

(ii) the lighthouse to the island

(iii) the island to the rocks

(iv) the rocks to the dock

(b) Write down the vector that would represent sailing directly from the dock to the island.

How does this relate to your answers in (a)(i) and (a)(ii)?

(c) Repeat part (b) for the total journey starting from the dock and returning back to the dock.

What is the overall vector that represents this journey?

4

(a) Write down two vectors that are parallel to the vector $\begin{pmatrix} 2 \\ 3 \end{pmatrix}$ and are in the same direction.

(b) Write down two vectors that are parallel to the vector $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$ and is in the opposite direction.

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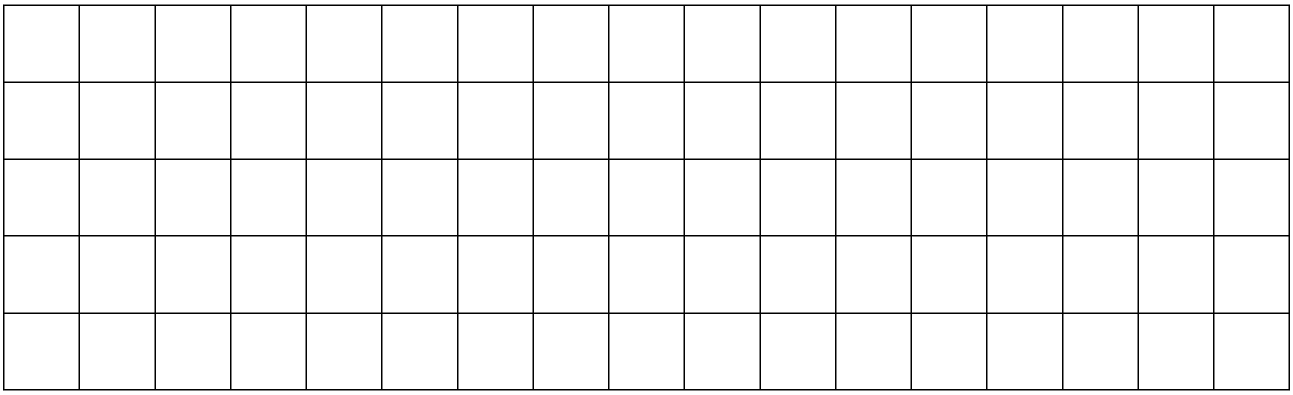
5 On the grid below draw a vector and write it in column vector form.

Now draw a vector perpendicular to the first vector and write it in column form.

What connection do you notice?



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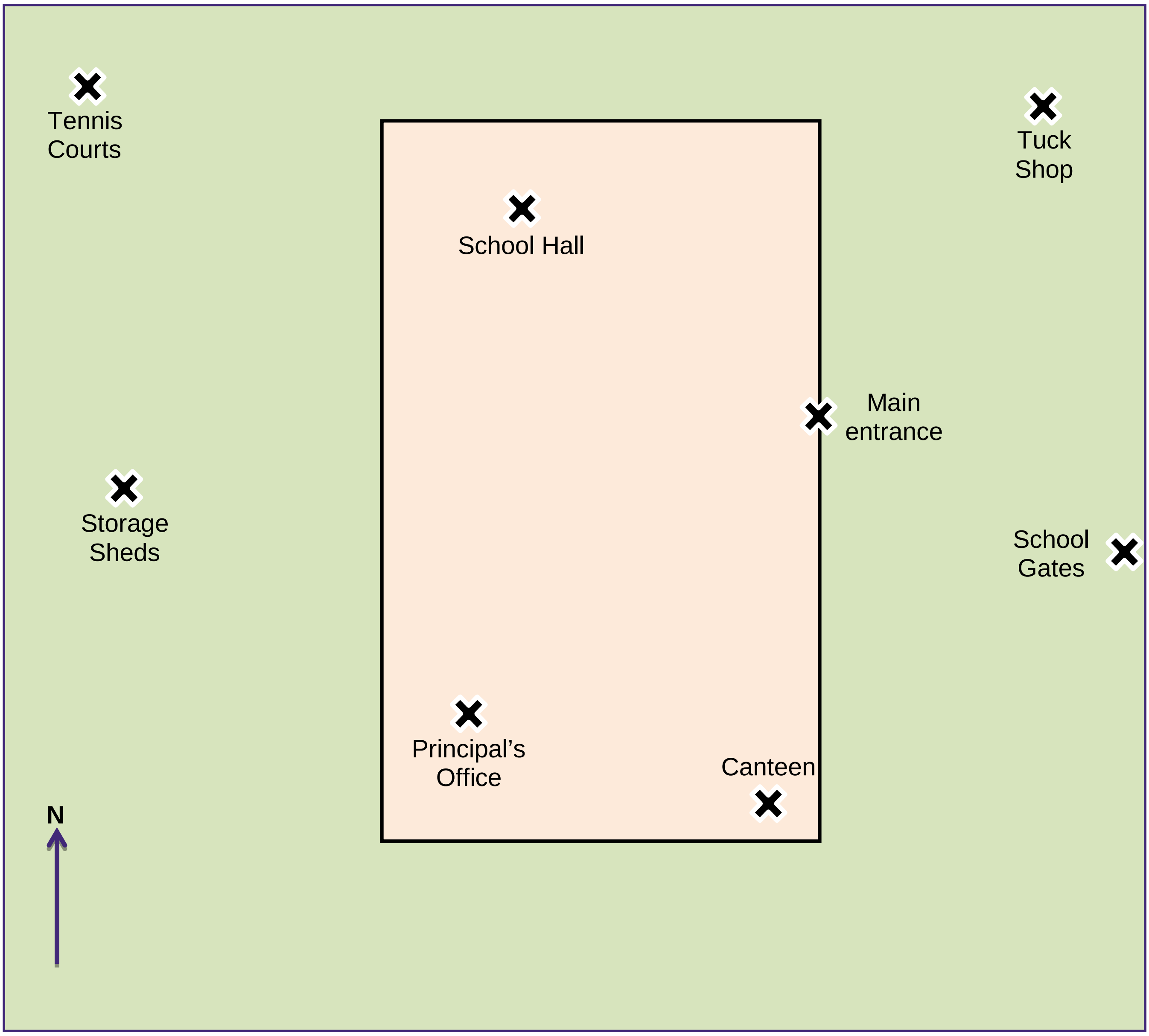
Skills Builder 2: Vectors and bearings

1 The map below shows some key points in a school and within a school's grounds.

The scale used is 1 cm to represent 100 m



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(a) The following statements describe the vector between various locations in the school grounds.

Complete the gaps in the statements:

(i) The Main entrance is ____________ from the tennis courts on a bearing of ____________

(ii) The ____________ is 1170 m from the Tuck Shop on a bearing of 202°.

(iii) To get from the ____________ to the Storage Sheds, walk ____________ on a bearing of 275°.

(iv) To reach the School Hall from the ____________, walk ____________ on a bearing of 005°.

(v) To walk from the School Hall to the School Gates via the Principal's Office, first walk

____________ on a bearing of ____________ and then walk ____________ on a bearing

of ____________

(b) A student arrives at the school gates at 8.53 am and runs to the Tuck Shop, where it takes him 1 minute to be served.

He then runs to the canteen to buy a drink, where he takes 1.5 minutes to be served.

He then runs to the School Hall, where assembly starts at 9 am.

If the student can run at a speed of 16 m/s, will he get to assembly on time?

State the bearing he must run on for each of the three stages of his run.

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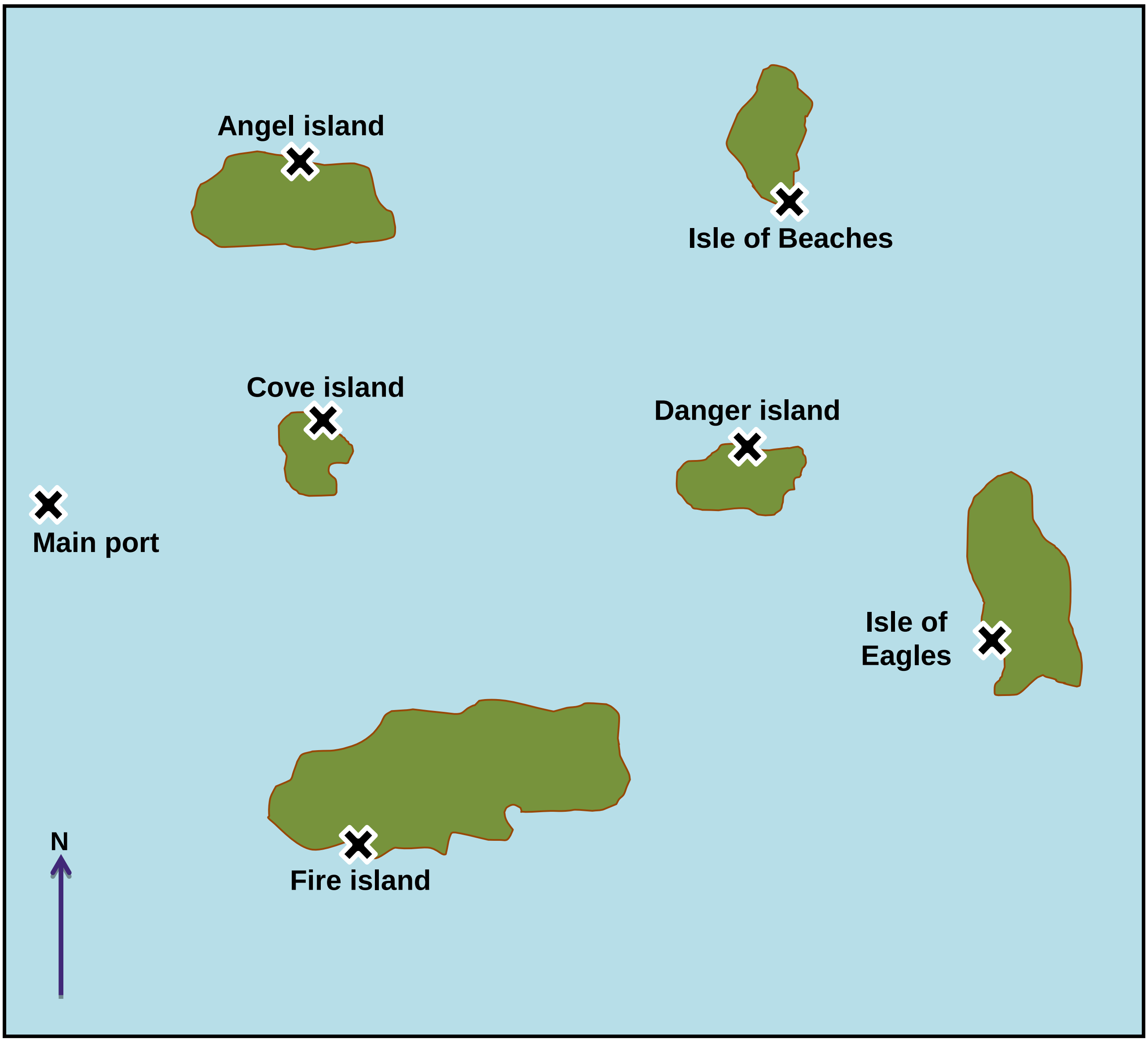
2 The map below shows the location of a number of islands in relation to the main port.

The scale used is 1 cm to represent 1 kilometre.



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(a) Fully describe the vectors that represent the following:

- **(i)** from Cove Island to the Isle of Eagles
- **(ii)** from Fire Island to the Main port
- **(iii)** from Danger Island to the Isle of Beaches
- **(iv)** from the Isle of Beaches to Danger Island
- **(v)** from the Isle of Eagles to Angel island

(b) The captain of a small cargo boat sails from the Main Port to the Isle of Eagles where she picks up some cargo to take to the isle of Beaches. After dropping off this cargo, she sails to Cove Island before returning to the Main Port.

- **(i)** Write down the directions for the route on which the captain should sail, giving bearings and distances.
- **(ii)** If the captain sets sail at 8 am and sails at an average speed of 35 km/h, what time will the boat arrive back to the Main Port?

  You may assume the loading and unloading of the boat and the other stop-offs total 2.5 hours.
- **(iii)** What assumption about the islands on the map has been made when carrying out these calculations?

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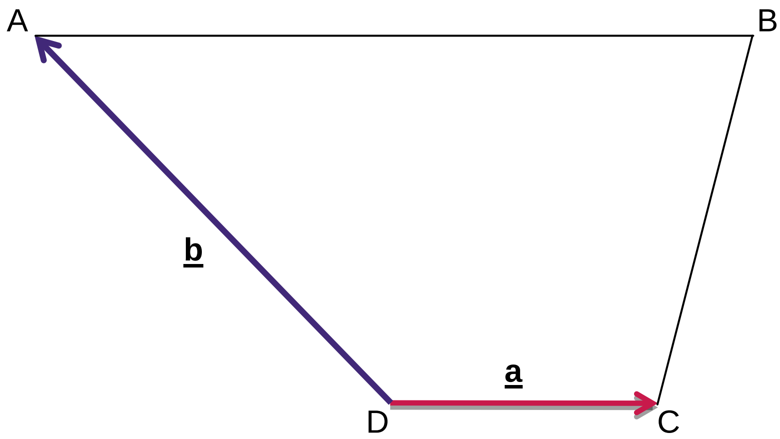
Skills Builder 3: Geometrical proof

1 In the diagram below AB is parallel to DC.



AB is three times as long as DC.

Side DC is represented by the vector **a** and side DA is represented by vector **b**.

(a) Write down the vector expressions for:

(i) $\overrightarrow{AB}$

(ii) $\overrightarrow{AC}$

(iii) $\overrightarrow{BC}$

Point M is the midpoint of side DA and N is the midpoint of DC.

(b) Write down the vector expressions for:

(i) $\overrightarrow{DM}$

(ii) $\overrightarrow{DN}$

(iii) $\overrightarrow{MN}$

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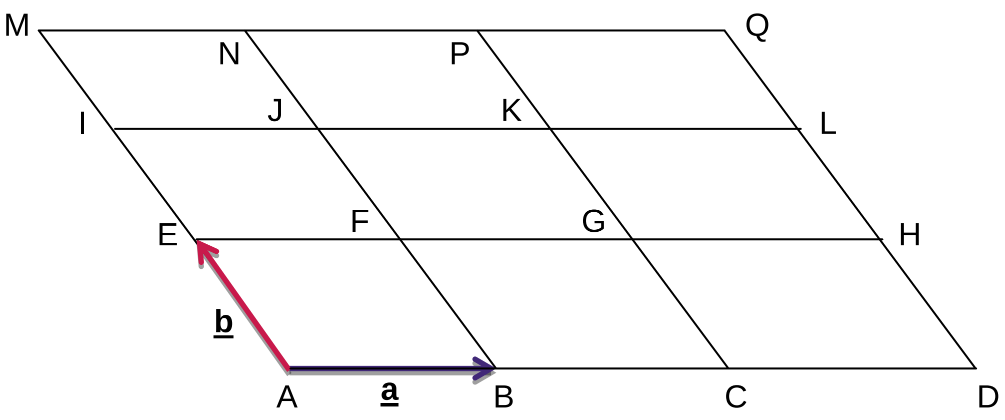
2 The grid below is made up of nine congruent parallelograms.

Side AB is represented by the vector **a** and side AE is represented by the vector **b**.



(a) Write down the expressions for the following vectors:

(i) $\overrightarrow{FG}$

(ii) $\overrightarrow{FB}$

(iii) $\overrightarrow{AF}$

(iv) $\overrightarrow{LA}$

(v) $\overrightarrow{MG}$

(b) Write down a vector in the form which are equivalent to:

(i) $2\underline{\mathbf{a}} - \underline{\mathbf{b}}$

(ii) $\underline{\mathbf{a}} + 3\underline{\mathbf{b}}$

(iii) $-2\underline{\mathbf{b}}$

(iv) $-(\underline{\mathbf{a}} + \underline{\mathbf{b}})$

(v) $\underline{\mathbf{b}} - 2\underline{\mathbf{a}}$

In part (b) there might be more than one possible answer

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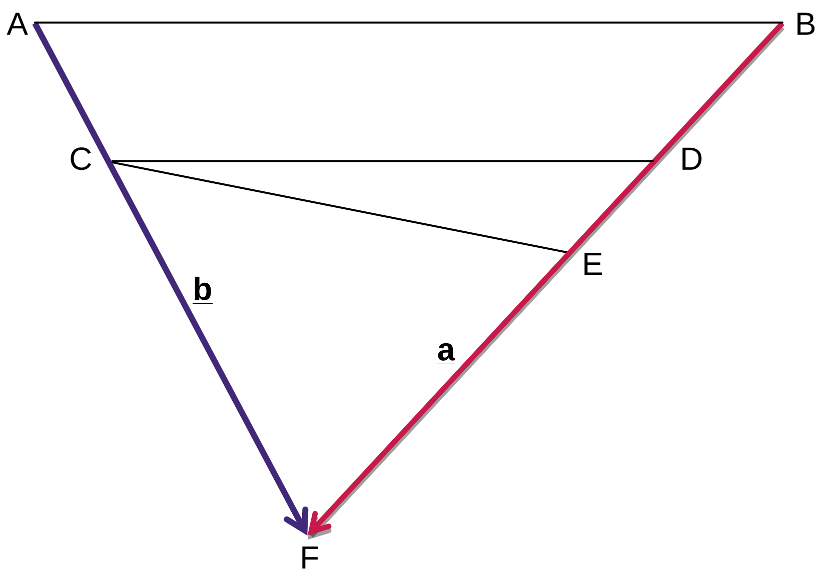
3 In the diagram below, AB and CD are parallel.

Point C lies one-third of the way along the side AF and point D lies one-third of the way along the side BF.

Lengths FE and EB are equal.

Vectors $\underline{\mathbf{a}}$ and $\underline{\mathbf{b}}$ are labelled on the diagram.



Write down expressions in terms of $\underline{\mathbf{a}}$ and $\underline{\mathbf{b}}$ for the following vectors.

Simplify your answers as far as possible.

(i)	$\overrightarrow{AC}$	**(ii)**	$\overrightarrow{BD}$
(iii)	$\overrightarrow{CF}$	**(iv)**	$\overrightarrow{DF}$
(v)	$\overrightarrow{BE}$	**(vi)**	$\overrightarrow{CE}$
(vii)	$\overrightarrow{DE}$	**(viii)**	$\overrightarrow{CD}$

Explain how your answer to part (viii) shows that AB and CD are parallel.

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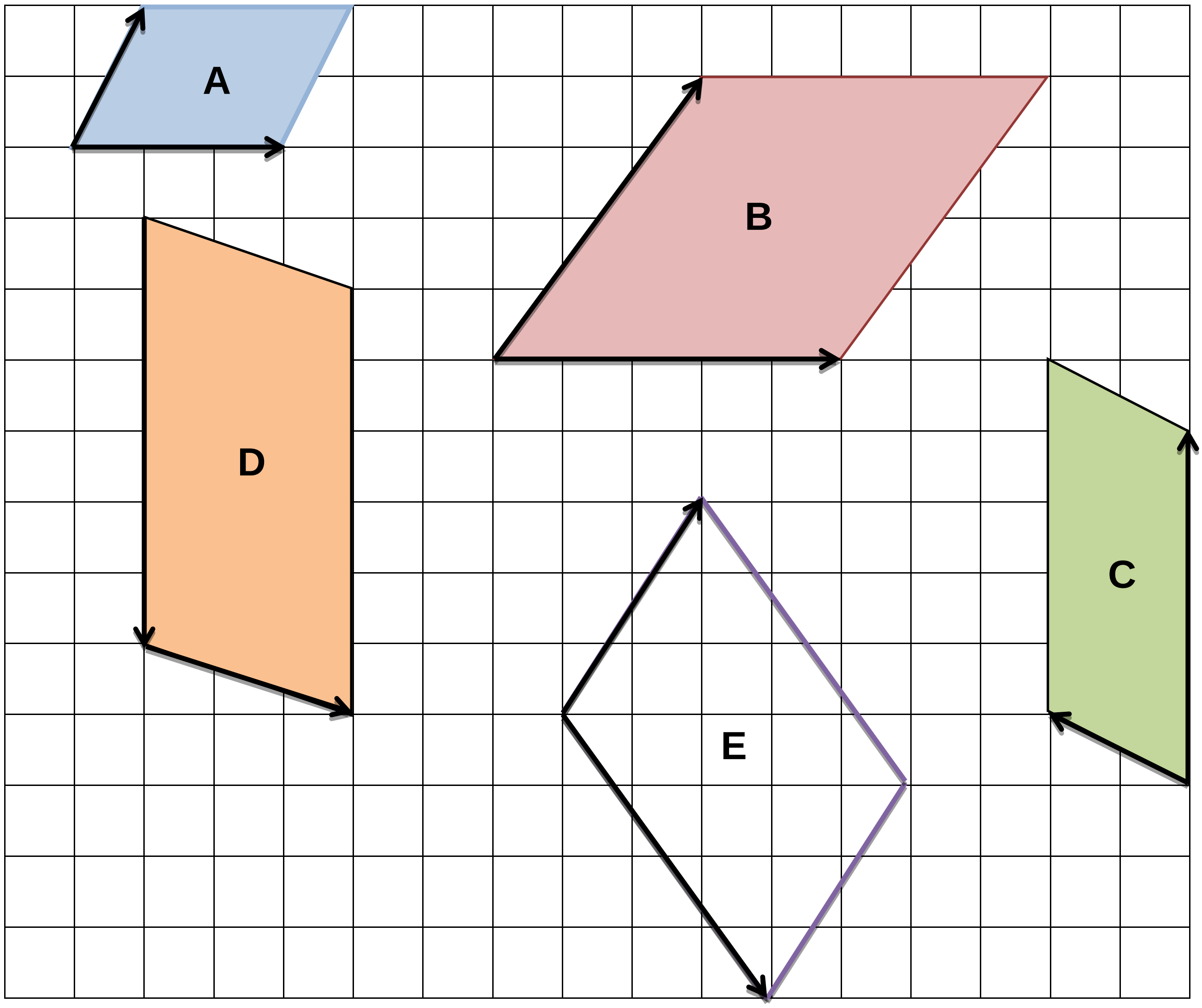
Problem solving 1: Vector Investigation

For each of the following parallelograms:

- write down the two vectors that make up the base and left hand side of the shape (labelled with arrows)
- find the area of the parallelograms

Is there a connection between the two vectors and the area of the parallelogram?



Investigate further by drawing some parallelograms of your own on squared paper.

Will your rule still work if the **a** side vector is drawn from right to left (as in parallelogram C)?

Or if the base vector is drawn from downwards rather than upwards? (as in parallelogram D)?

Does the base of the parallelogram have to be horizontal? Must the side be vertical? (see parallelogram E).

Can you find an algebraic rule?

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Problem solving 2: Vector Loop Cards

Teacher Instructions

The vector loop cards can either

- be enlarged and displayed around the classroom to form a ‘treasure hunt’ style activity

  or

- each pair of students can be given a set of cards

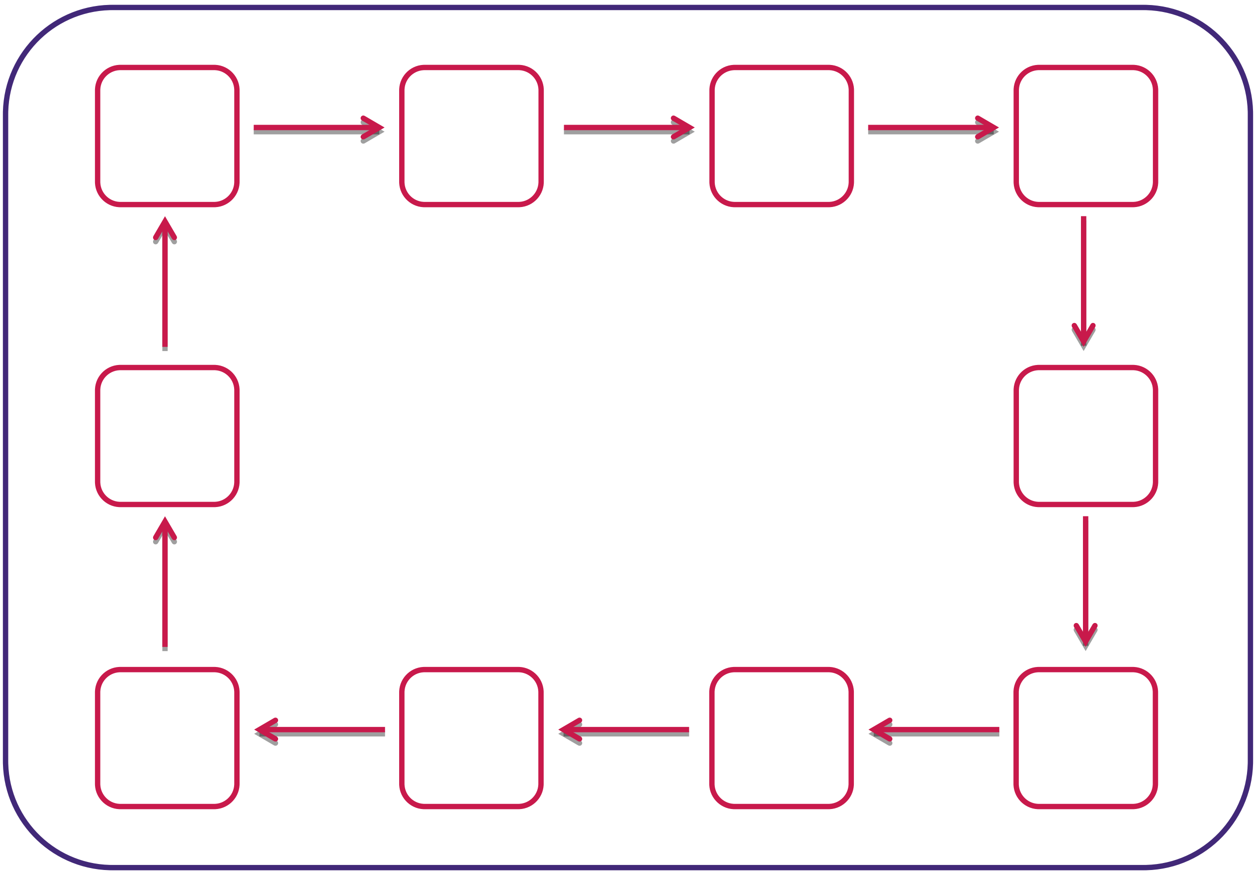
Students will also need a copy of the answer grid and cm squared paper for rough work. For most questions a scale diagram will be needed, and a scale of 1cm for 1km would be useful for these diagrams.

Game Instructions

Start with any card and work out the answer to the question. The answer will appear on one of the other cards which should be answered next.

Continue to answer questions in the loop until you reach the answer on the first card chosen.

The letter on each card should be written on the answer grid (below) after each question is answered.



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A

Answer

$$\begin{pmatrix} -1 \\ -10 \end{pmatrix}$$

Question

From A, travel 7 km due East to B and then 3 km on a bearing of 200° to C.

The direct vector from A to C would be…?

B

Answer

$$\begin{pmatrix} -2 \\ -5 \end{pmatrix}$$

Question

From A, travel 6.5 km on a bearing of 310° to B and then 9 km on a bearing of 105° to C.

The direct vector from C to A would be…?

C

Answer

$$\begin{pmatrix} -3 \\ 1 \end{pmatrix}$$

Question

To travel from the Supermarket to the Post Office, Fred would need to drive 3 km on a bearing of 160°.

To drive from the Post Office to the Supermarket, Fred would need to drive along the vector…?

D

Answer

4 km on a bearing of 245°

Question

A road from town A to town B is along the vector $\begin{pmatrix} -7 \\ -4 \end{pmatrix}$ km.

A motorway is to be built perpendicular to this road.

The motorway would be in the direction of the vector …?

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E

Answer:

3.6 km due South

Question:

Gemma walks from her house 8.6km on a bearing of 315° then walks along the vector $\begin{pmatrix} 7 \\ 4 \end{pmatrix}$ km.

To return to her house she must walk along the vector …?

F

Answer:

$\begin{pmatrix} 4 \\ -7 \end{pmatrix}$

Question:

A yacht sails from a harbour 10 km on a bearing of 080° then turns and sails 10 km on a bearing of 280°.

The vector the yacht must sail along to return to the harbour is…?

G

Answer:

6 km due North

Question:

A rectangle has two sides that can be represented by the vector $\begin{pmatrix} 5 \\ 5 \end{pmatrix}$

A vector that could represent the other two sides is…?

H

Answer:

6.6 km on a bearing of 115°

Question:

A boat sails from a dock along the vector $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$ km and then sails along the vector $\begin{pmatrix} -3 \\ -2 \end{pmatrix}$ km.

To return to the dock, sail along the vector….?

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I

Answer:

3 km on a bearing of 340°

Question:

The vector between Skull Island and Crossbone Creek is $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$.

The vector between Crossbone Creek and Danger Dock is $\begin{pmatrix} -5 \\ -1 \end{pmatrix}$

The vector from Skull Island to Danger Dock is…?

J

Answer:

$\begin{pmatrix} 3 \\ -3 \end{pmatrix}$

Question:

I park my car and walk 5 km North-East and then walk 4.7 km on a bearing of 187°

The vector that I should walk on to take me back to my car is…?

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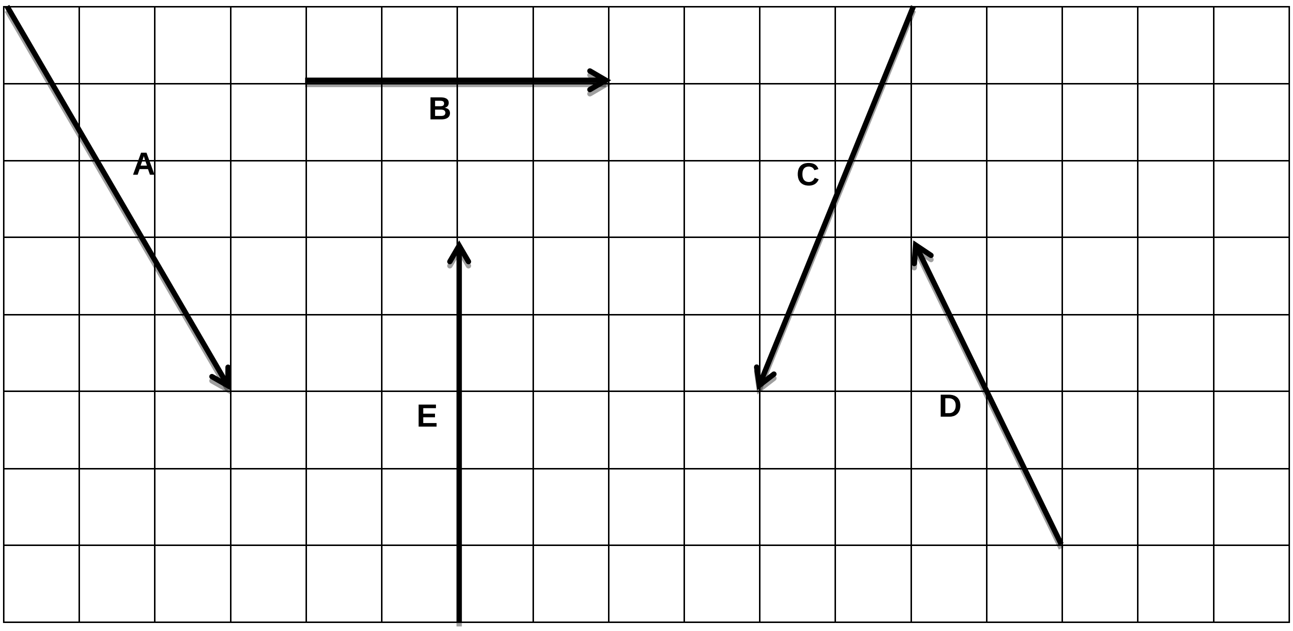
Answers

Skills builder 1: Column Vectors

1 A = $\begin{pmatrix} -2 \\ 5 \end{pmatrix}$ B = $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$ C = $\begin{pmatrix} -2 \\ -1 \end{pmatrix}$ D = $\begin{pmatrix} 4 \\ 0 \end{pmatrix}$ E = $\begin{pmatrix} -6 \\ -1 \end{pmatrix}$ F = $\begin{pmatrix} 2 \\ 5 \end{pmatrix}$ 6 = $\begin{pmatrix} 0 \\ -7 \end{pmatrix}$

2



3 (a) (i) $\begin{pmatrix} 5 \\ 4 \end{pmatrix}$

(ii) $\begin{pmatrix} 9 \\ -2 \end{pmatrix}$

(iii) $\begin{pmatrix} -5 \\ -4 \end{pmatrix}$

(iv) $\begin{pmatrix} -9 \\ 2 \end{pmatrix}$

(b) $\begin{pmatrix} 14 \\ 2 \end{pmatrix}$ This is the same as adding together the vectors (a)(i) and (a)(ii),

ie 14 = 5 + 9 and 2 = 4 + –2

(c) The vector for the whole journey is $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$ which is the same as adding together all four vectors from (a)(i) - (a)(iv)

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4 (a) Any two vectors that are of the form $\begin{pmatrix} 2n \\ 3n \end{pmatrix}$ where n is positive

(b) Any two vectors that are of the form $\begin{pmatrix} -3b \\ 4n \end{pmatrix}$ where n is positive

5 The vector $\begin{pmatrix} a \\ b \end{pmatrix}$ has perpendicular vector $\begin{pmatrix} -b \\ a \end{pmatrix}$ or $\begin{pmatrix} b \\ -a \end{pmatrix}$ For example, the vector $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ has a perpendicular vector $\begin{pmatrix} -1 \\ 2 \end{pmatrix}$ or $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

Skills builder 2: Vectors using bearings

1 (a) (i) The Main entrance is **1250 m** from the tennis courts on a bearing of .

(ii) The **Canteen** is 1170 m from the Tuck Shop on a bearing of 202°.

(iii) To get from the **School Gates** to the Storage Sheds, walk **1560 m** on a bearing of 275°.

(iv) To reach the School Hall from the **Principal's Office**, walk **780 m** on a bearing of 005°.

(v) To walk from the School Hall to the School Gates via the Principal's Office, first walk **780 m** on a bearing of **185°** and then walk **1050 m** on a bearing of **076°**.

(b) From the School Gates to the Tuck Shop is 700 m on a bearing of 350°; from the Tuck Shop to the Canteen is 1170 m on a bearing of 202°; and the School Hall is 1000 m on a bearing of 338°.

The total distance he runs is 2870 m, which would take 179.375 seconds, or just under 3 minutes. The time to get served is 2.5 minutes in total and so the student will arrive at the School Hall at 8.58 and 30 seconds. So yes, he will be on time for assembly.

2 (a) (i) From Cove Island to the Isle of Eagles - 10.6 km on a bearing of 109°.

(ii) From Fire Island to the Main Port - 7 km on a bearing of 317°.

(iii) From Danger Island to the Isle of Beaches - 3.8 km on a bearing of 010°.

(iv) From the Isle of Beaches to Danger Island - 3.8 km on a bearing of 190°.

(v) From the Isle of Eagles to Angel island - 12.7 km on a bearing of 305°.

(b) Sail from the Main Port for 14.4 km on a bearing of 100° to reach the Isle of Eagles. From there, collect the cargo for the Isle of Beaches and sail 7.4 km on a bearing of 337° to drop off the cargo. Then sail 7.8 km 245° to Cove Island and finally return to the Main Port by sailing on a bearing of 252° for a distance of 4.3 km.

The total sailing distance is 33.9 km which would take 0.9685… hours or approximately 58 minutes. Therefore, including stop-offs and loading/unloading the boat will arrive back in port at 11.28 am.

These calculations assume that the line of sailing is direct and that the port/harbour on each island lies on this direct line of sailing. In reality, it is likely that the boat would have to sail around the coast of an island to the place where it can dock. This is likely to increase the sailing distance and time.

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Skills builder 3: Geometrical proof

1 (a) (i) $3\underline{\mathbf{a}}$

(ii) $-\underline{\mathbf{b}} + \underline{\mathbf{a}}$ $\quad \underline{\mathbf{a}} - \underline{\mathbf{b}}$

(iii) $-3\underline{\mathbf{a}} - \underline{\mathbf{b}} + \underline{\mathbf{a}}$ or $-2\underline{\mathbf{a}} - \underline{\mathbf{b}}$

(b) (i) $\frac{1}{2}\underline{\mathbf{b}}$

(ii) $\frac{1}{2}\underline{\mathbf{a}}$

(iii) $-\frac{1}{2}\underline{\mathbf{b}} + \frac{1}{2}\underline{\mathbf{a}}$ or $\frac{1}{2}(\underline{\mathbf{a}} - \underline{\mathbf{b}})$

2 (a) (i) $\underline{\mathbf{a}}$

(ii) $-\underline{\mathbf{b}}$

(iii) $\underline{\mathbf{a}} + \underline{\mathbf{b}}$

(iv) $-3\underline{\mathbf{a}} - 2\underline{\mathbf{b}}$

(v) $2\underline{\mathbf{a}} - 2\underline{\mathbf{b}}$

(b) Example answers are given for each part of this question. Any parallel vector is correct for each part.

(i) $\overrightarrow{EC}$

(ii) $\overrightarrow{BP}$

(iii) $\overrightarrow{KC}$

(iv) $\overrightarrow{GC}$

(v) $\overrightarrow{GI}$

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(a) (i) $\frac{1}{3}\underline{\mathbf{b}}$

(ii) $\frac{1}{3}\underline{\mathbf{a}}$

(iii) $\frac{2}{3}\underline{\mathbf{b}}$

(iv) $\frac{2}{3}\underline{\mathbf{a}}$

(v) $\frac{1}{2}\underline{\mathbf{a}}$

(vi) $\frac{2}{3}\underline{\mathbf{b}} - \frac{1}{2}\underline{\mathbf{a}}$

(vii) $\frac{1}{6}\underline{\mathbf{a}}$

(viii) $\frac{2}{3}\underline{\mathbf{b}} - \frac{2}{3}\underline{\mathbf{a}}$

Vector $\overrightarrow{AB}$ is $\underline{\mathbf{b}} - \underline{\mathbf{a}}$ and $\overrightarrow{CD}$ can be written $\frac{2}{3}(\underline{\mathbf{b}} - \underline{\mathbf{a}})$. This is a multiple of $\underline{\mathbf{b}} - \underline{\mathbf{a}}$ and so the two vectors are parallel.

Problem Solving 1: Vector investigation

A vectors $\begin{pmatrix}3\\0\end{pmatrix}$ and $\begin{pmatrix}1\\2\end{pmatrix}$ area 6

B vectors $\begin{pmatrix}5\\0\end{pmatrix}$ and $\begin{pmatrix}3\\4\end{pmatrix}$ area 20

C vectors $\begin{pmatrix}0\\5\end{pmatrix}$ and $\begin{pmatrix}-2\\1\end{pmatrix}$ area 10

D vectors $\begin{pmatrix}3\\-1\end{pmatrix}$ and $\begin{pmatrix}0\\-6\end{pmatrix}$ area 18

E vectors $\begin{pmatrix}2\\3\end{pmatrix}$ and $\begin{pmatrix}3\\-4\end{pmatrix}$ area 17

For a general parallelogram with vectors $\begin{pmatrix}p\\q\end{pmatrix}$ and $\begin{pmatrix}r\\s\end{pmatrix}$ the area is obtained by calculating $ps - rq$. If this is negative, remove the negative sign.

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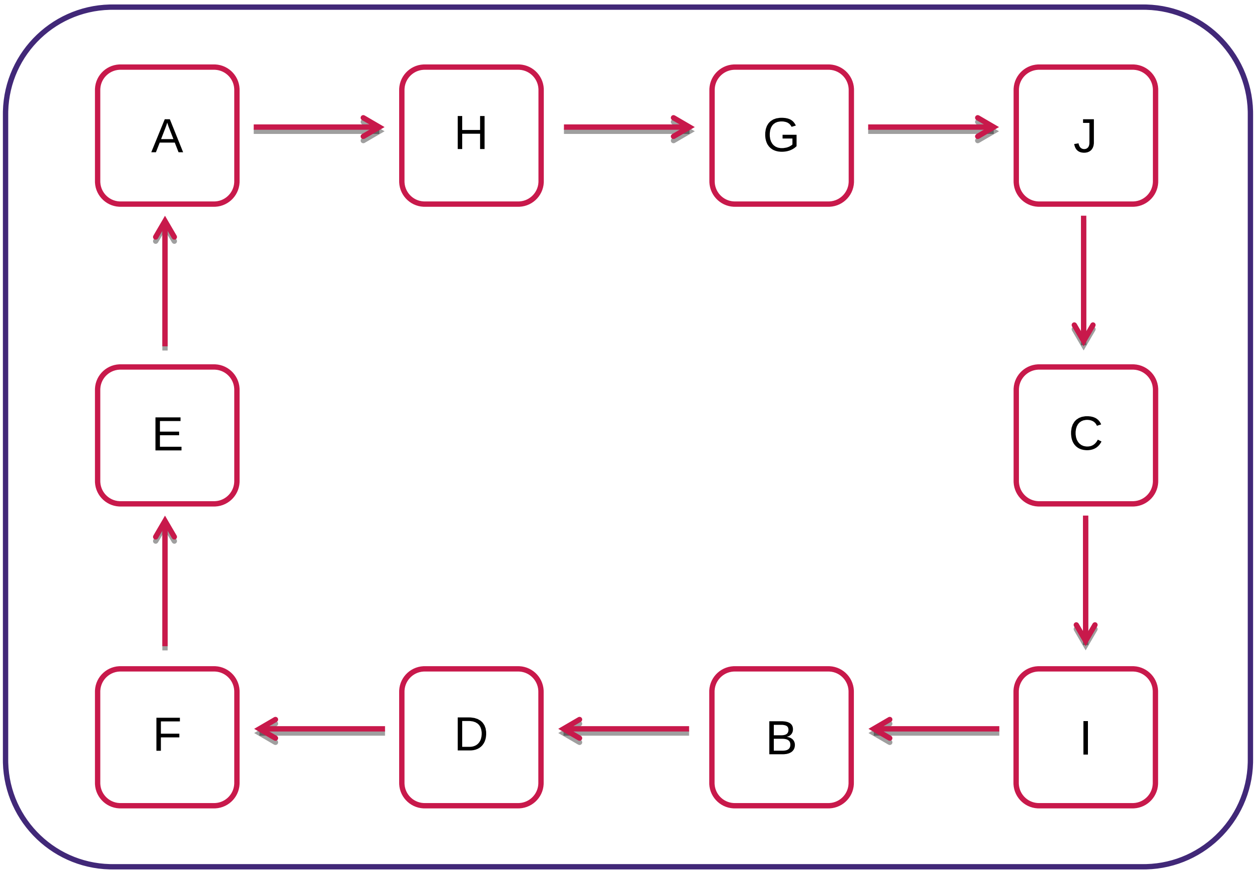
Problem solving 2 : Vector Loop Cards



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